Fluid Flow Through Pipes
SDS 374C Final Project

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1 Introduction and Purpose

The topic I chose to research for this project was fluid flow. Originally I had intended to simulate the results from a laboratory experiment I had completed earlier this semester by modeling the flow of fluid around a cylinder. However, I experienced a lot of difficulty simulating basic scenarios by solving the Navier Stokes equations, and instead decided to study the flow of fluid through a channel. While it may seem trivial, understanding this flow is essential to modern system design, and was the main focus of my fluid mechanics course in the latter half of the semester. The key aspects of interest are the pressure and velocity components as the flow travels, along with studying the viscous losses associated with the flow.

To learn the basics of computational fluid mechanics I read several textbooks and online articles. However, the most useful resource by far was the 12 Steps to Navier-Stokes online course published by the Lorena A. Barba research lab. Through a series of interactive ipython notebooks they model several types of one dimensional and two dimensional flows, before eventually building up to a basic solution of the Navier-Stokes equations. This resource showed me a relatively simple was to model fluid flow, and provided the equations necessary along with a rough outline of the different steps required to solve them.

2 Algorithim to be Parallelized

To solve this problem the two dimensional Navier Stokes equations were used in addition to the Possion Pressure equation. These are all second order differential equations and will have to be approximated discretely. The original equations are depicted below, and offer a comprehesive description of Newtonian viscous flow. Additionally, if the flow is assumed to be incompressible, (an assumption which is valid for flows with a Mach number slower than 0.3) then the Possion Pressure equation can be used as a continunity constraint. Together these three equtations allow for the pressure along with the horizontal and veritical velocity at each point to be known.

Navier Stokes X-Momentum

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)$$

Navier Stokes Y-Momentum

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \nu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right)$$

Possion Pressure Equation

$$\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} = -\rho\left(\frac{\partial u}{\partial x}\frac{\partial u}{\partial x} + 2\frac{\partial u}{\partial y}\frac{\partial v}{\partial x} + \frac{\partial v}{\partial y}\frac{\partial v}{\partial y}\right)$$

Due to viscous effects fluid travelling in a channel or pipe develops a head loss, and need some force to drive the flow. Therefore a constant force was added to the X-Momentum equation. Once these equations are discretized they can be rearranged to solve for the next time step for the horizontal and vertical velocity. This can be done because the momentum equations have a component of velocity that is differentiated by time. However, in the pressure equation the pressure terms are only differentied by the x and y dimensions. Therefore, the pressure equation needs to be solved by successive iterations, while the momentum equations can be solved in a single timestep.

3 How that Parallelization was acheived, design goals

The continous flow region was divided into a grid composed of several uniform cells. Each of these cells have their own horizontal and vertical velocity components and along with a pressure value. These cells can be solved independently within each time-step, and are thereby suitable for parallelization using MPI. The solved values for each cell are distributed among the different processors used. To solve each cell the values of it's neighboors need to be known, so there has to be some communication at the end of each time step. These communicated values are then used in the discritized equations to solve for the values of each cell at the next time step. This continues for the specified amount of time and for each cell in the grid. The cells are divided up linearly and assigned to a processor. If the number of cells are not evenly divisable, then the remaining cells are added to seperate processors one at a time. By doing so the most unequal workload would be one addtional cell on some processors. If each processor has several hundred cells already, this imbalance is neglible.

The algorithim stated in the previous section was broken into two main stages; the pressure solving and the velocity solving. The pressure solution for each cell needs to be done in pseudo-time, with communication between processors at the end of each pseudo-time step. However, only the previous pressure components actually change with each pseudo-time step. The other components of the pressure equation are all due to the velocity, which doesn't change during the pseudo-time steps. Therefore the pressure equation can be broken into two parts. The velocity component's contribution to the pressure is solved at the begining of each full time step, while the pressure components are modified within the pseudo-time loop. At the end of each pseudo-time step these two pieces are added together to find the pressure for the current cell. By moving the constant velocity components out of the pseudo-time loop the total computation for each pseudo-time step is cut in half. Once all of a processor's cells have been solved for then an all gather vector communication is performed. The all gather collects each of the data from the processors local cells, then broadcasts it to all of the other processors and stores in their individual global arrays. This solves the issue of distributed data for each cell, allowing them to know the values of their neighboors for their individual calculations. An all gather vector was needed here instead of a regular all gather because of the potential unequal loading of the processors. The number of cells on each processor is found initially and broadcast to every processor so the variable length communication can take place.

Once the pressure for each cell is found the velocity components can be found in a similiar fashion. The velocity components only require one time step to find a solution however, instead of iterating over pseudo-time steps in addition to the actual time steps. Using the pressure and velocity found for the previous time step the X and Y components of velocity for the current step can be found. This is then stored along with the previously found pressure for this time step in an multi-dimensional array of structs. Once this is done for each cell on the processor another vector all gather is performed, distributing the data from each cell to an array in each processor. Once this is done the time step is incrimented, and the pressure and velocity data for each cell is found again. This continues until the last time step is reached, after which the data is written out sequentially to an HDF5 file for post-processing visualization. A pseudo-code representation of this parallelization is shown below.

```
while counter < numberTimeSteps
        //Make initial pressure array and
        //Find velocity component of pressure
        for i in numCells
                solvedPressure[0][i] = solvedData[counter-1][i].
                   p
                velocityComponent[i] = presVelComponent(
                   xLocation,yLocation)

        //Find pressure
        while subCounter < numberPseudoTimeSteps
                for i in numCells
                        localPresData[i] = pressSolve(xLocation,
                           yLocation)

                MPI_Allgatherv(localPresData, solvedPrePresData[
                   subCounter])

                subCounter += 1;

        //Find velocity and store data
        for i in numCells
                localData[i].u = xMomentumSolve(xLocation,
                   yLocation)
                localData[i].v = yMomentumSolve(xLocation,
                   yLocation)
                localData[i].p = solvedPressure[
                   numberPseudoTimeSteps-1][startingLocation+i]

        MPI_Allgatherv(localVelData,  solvedVelData[counter][0])
        subCounter = 1
        counter += 1
```

4 Timing Results and Trace

The program was run for a 300x300 size grid with 500 time steps and 300 pseudotime steps. The timing results for this large-scale run can be seen in Figure 1. As one can see, the run time actually increases with the number of cores initially. The reason behind this can be seen in the trace diagram for a small-scale version of this program in Figure 2. One can see the large blocks of MPI communication which fills up most of the diagram. The tiny slivers of brown green in between them are where the actual calculations are being performed. Therefore the timing behavior observed makes intuitive sense. For large work loads the performance is increased as the number of processors increase, to a certain extent. Once the workload on each processor is light enough the communication between the processors becomes a much more limiting factor than the calculations itself. After this point, additional processors will do nothing fill the network with more communications and make the program more and more reliant on slow communications to do the same amount of work. This explains why the performance gain per processor isn't a linear or even asymptopic curve, but rather can actually increase the run time of the program. However, the run time then drastically decreases as more processors are added, which is not the expected behavior at all and I don't understand. The efficiency is very low though, seeing as how with 80 processors the program runs slower than 5 processors.

In Figure 3 one can see the solved image of flow for laminar flow through a channel. The constant color background means that the pressure is constant everywhere, and the viscous effects due to the walls clearly reduce the energy of the flow. Because the velocity gradient isn't a parabola this means that the flow is still developing, and viscous effects will become more dominant as time progresses. All of these observations coincide exactly with the ideal and experiemental behavior of two dimensional flow through a pipe, thus proving that the model was successful in modelling flow under these conditions.

Figure 1: Timing results, Number of Processors vs. Runtime (s)

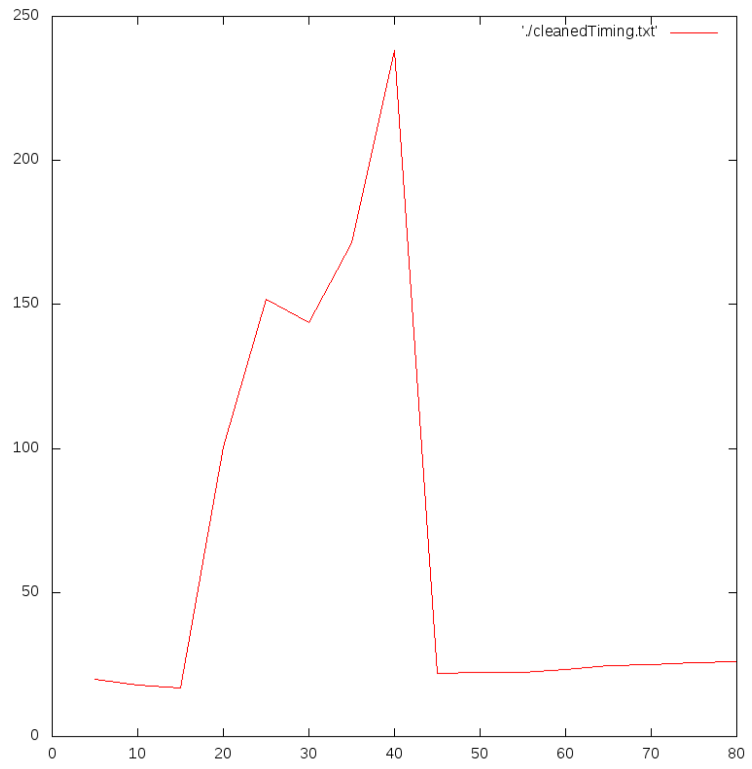


Figure 2: TAU trace of the program

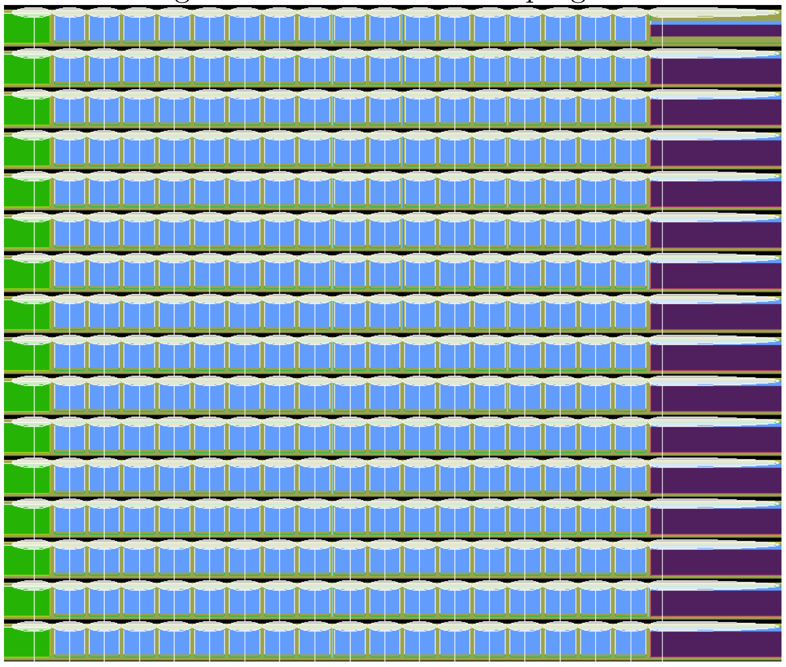
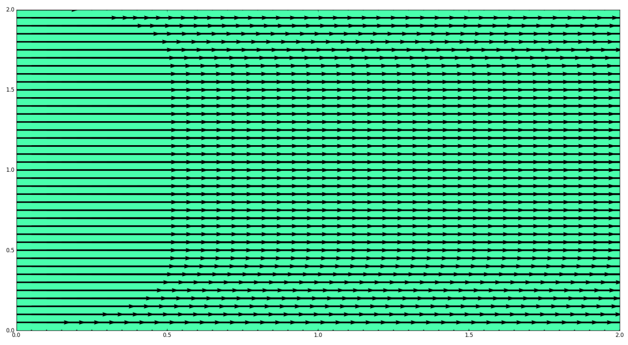


Figure 3: The solved velocity and pressure data


2.0
1.5
1.0
0.5
0.0
0.0
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1.5
2.0